Fall term 2018

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Topology I

Sheet 14

Exercise 1.

a) Prove that if $f : S^n \longrightarrow S^n$ is a homotopy equivalence, then $\deg(f) = \pm 1$.

b) Give examples of homeomorphisms realizing both values for the degree.

Exercise 2. Let $a : S^n \longrightarrow S^n$ be the antipodal map, i.e. given by $p \longmapsto -p$.

a) Compute $\deg(a)$.

b) Prove that a is homotopic to Id_{S^n} if, and only if, n is odd.

Exercise 3. Let $f : S^n \longrightarrow S^n$ be a continuous map. Show that

a) if f has no fixed point, then $\deg(f) = (-1)^{n+1}$.

b) if f has the property that $f(p) \neq -p$ for all $p \in S^n$, then $\deg(f) = 1$.

Exercise 4.

a) Let

$$0 \longrightarrow C_\bullet \xrightarrow{f} D_\bullet \xrightarrow{g} E_\bullet \longrightarrow 0$$

be a short exact sequence of chain complexes. In the lecture it was almost shown that this yields a long exact sequence of the form

$$... \longrightarrow H_k(D_\bullet) \xrightarrow{g_*} H_k(E_\bullet) \xrightarrow{\partial} H_{k-1}(C_\bullet) \xrightarrow{f_*} H_{k-1}(D_\bullet) \xrightarrow{g_*} ...$$

Finish the proof by showing that $\ker f_* = \mathrm{im}(\partial)$.

b) (The five lemma). Given two exact sequences of abelian groups of the form

$$A_i \longrightarrow B_i \longrightarrow C_i \longrightarrow D_i \longrightarrow E_i$$

$(i = 1, 2)$ and a map φ from the first sequence to the second which is an isomorphism between the B_i and D_i, surjective from A_1 to A_2 and injective from E_1 to E_2. Show that φ is an isomorphism between C_1 and C_2.

Hand in: during the lecture on Tuesday, February 5th.