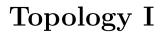


LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Fall term 2018

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Sheet 14

Exercise 1.

- a) Prove that if $f: S^n \longrightarrow S^n$ is a homotopy equivalence, then $\deg(f) = \pm 1$.
- b) Give examples of homeomorphisms realizing both values for the degree.

Exercise 2. Let $a: S^n \longrightarrow S^n$ be the antipodal map, i.e. given by $p \longmapsto -p$.

- a) Compute $\deg(a)$.
- b) Prove that a is homotopic to Id_{S^n} if, and only if, n is odd.

Exercise 3. Let $f: S^n \longrightarrow S^n$ be a continuous map. Show that

- a) if f has no fixed point, then $\deg(f) = (-1)^{n+1}$.
- b) if f has the property that $f(p) \neq -p$ for all $p \in S^n$, then $\deg(f) = 1$.

Exercise 4.

a) Let

$$0 \longrightarrow C_{\bullet} \xrightarrow{f} D_{\bullet} \xrightarrow{g} E_{\bullet} \longrightarrow 0$$

be a short exact sequence of chain complexes. In the lecture it was almost shown that this yields a long exact sequence of the form

$$\dots \longrightarrow H_k(D_{\bullet}) \xrightarrow{g_*} H_k(E_{\bullet}) \xrightarrow{\partial} H_{k-1}(C_{\bullet}) \xrightarrow{f_*} H_{k-1}(D_{\bullet}) \xrightarrow{g_*} \dots$$

Finish the proof by showing that ker $f_* = \operatorname{im}(\partial)$.

b) (The five lemma). Given two exact sequences of abelian groups of the form

$$A_i \longrightarrow B_i \longrightarrow C_i \longrightarrow D_i \longrightarrow E_i$$

(i = 1, 2) and a map φ from the first sequence to the second which is an isomorphism between the B_i and D_i , surjective from A_1 to A_2 and injective from E_1 to E_2 . Show that φ is an isomorphism between C_1 and C_2 .

Hand in: during the lecture on Tuesday, February 5th.