Fall term 2018

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Topology I

Sheet 13

Exercise 1. Let X and Y be topological spaces and $x \in X$ (resp. $y \in Y$) such that there exists a contractible neighbourhood $x \in U \subset X$ (resp. $y \in V \subset Y$). Compute the homology groups of the one point union $X \vee Y = X \sqcup Y/x \sim y$.

Exercise 2. Compute the homology groups of $S^n \times S^m$ for all $n, m \in \mathbb{N}$.
[**Hint**: Use Mayer-Vietoris to reduce to the computation of $H_i(S^1 \times S^1)$.]

Exercise 3. Suppose that $K \subset S^3$ is homeomorphic to S^1 and that there is an open neighbourhood U of K homeomorphic to $S^1 \times D^2$ so that K maps to $S^1 \times \{0\}$. Denote by $V = S^3 \setminus K$ the complement of K in S^3.

a) Compute the homology groups $H_i(V)$ (at least for $i \neq 2, 3$).

b) What can you say about $\pi_1(V)$?

Exercise 4. Let D^n be an open ball in $\mathbb{R}^n$. Show that if there is a homeomorphism $f : D^n \to D^m$, then $n = m$.

Exercise 5. Let $K = I \times I / \sim$ be the Klein bottle, where $(0, y) \sim (1, 1 - y)$ and $(x, 0) \sim (x, 1)$. Compute the homology groups $H_i(K)$ for all i.

Hand in: during the lecture on Tuesday, January 29th.