LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT

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Prof. D. Kotschick
Dr. J. Stelzig
G. Placini

# Topology I

Sheet 9

**Exercise 1.** Let $X, U, V$ be as in Seifert-Van Kampen theorem. Show that, if $U \cap V$ is simply connected, $\pi_1(X)$ is determined (up to isomorphism) by the following universal property:
For any group $G$ and pair of homomorphisms $\varphi_1 : \pi_1(U) \to G$ and $\varphi_2 : \pi_1(V) \to G$ there exists a unique homomorphism $\psi : \pi_1(X) \to G$ such that $\varphi_1 = \psi \circ j_1$ and $\varphi_2 = \psi \circ j_2$.
[Remark: Such a group is called the **free product** of $\pi_1(U)$ and $\pi_1(V)$.]

**Exercise 2.** Let $X$ be a connected finite one dimensional CW-complex.

a) Find a maximal tree in X ( i.e. a CW-complex which becomes disconnected whenever one removes a point in a 1-cell).

b) Show that this tree is contractible.

c) Show that $X$ is homotopy equivalent to a CW complex with exactly one zero cell and deduce that the fundamental group of $X$ is a free group.

**Exercise 3.**

a) Let $X$ be a connected finite one dimensional CW-complex which by the previous exercise is homotopy equivalent to the one point union of $n$ circles. Show that the number of 0-cells $v$ and the number of 1-cells $e$ satisfy the relation $n = e - v + 1$.

b) Prove that if $G$ is a free group on $n$ generators and $H$ is a subgroup of finite index $d$, then $H$ is a free group with $dn - d + 1$ generators.

**Exercise 4.** Let $X$ be a CW-complex and $f : S^{n-1} \to X$ a continuous map. Show that if $n > 2$ then $\pi_1(X) = \pi_1(Y)$ where $Y = X \cup_f e^n$ is obtained from $X$ by attaching a $n$-cell with attaching map $f$.

Hand in: during the lecture on Tuesday, December 18th.