Fall term 2018

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Topology I

Sheet 8

Exercise 1. A section of a covering $p : X \longrightarrow Y$ is a continuous mapping $s : Y \longrightarrow X$ such that $p \circ s$ is the identity. Show that if a G-covering has a section, then the covering is trivial as a G-covering.

Exercise 2. Let Y be a connected topological space, $p : X \longrightarrow Y$ be a G-covering and φ_1, φ_2 be automorphisms of the G-covering $p : X \longrightarrow Y$. Show that if φ_1, φ_2 agree on one point, then $\varphi_1 = \varphi_2$.

Exercise 3. Let $m, n \in \mathbb{Z}_{>1}$ and $\mu_n := \{\zeta \in \mathbb{C} \mid \zeta^n = 1\}$ the group of n-th roots of unity. Construct an action of μ_n on S^{2m-1} such that the quotient has fundamental group μ_n. Deduce that every group G which is of the form $G = \mathbb{Z}^k \oplus \bigoplus_{i=1}^{j} \mathbb{Z}/r_i\mathbb{Z}$ for some $k, r_1, ..., r_j \in \mathbb{Z}_{>0}$ is the fundamental group of a topological space.

Exercise 4. Let $p : X \longrightarrow Y$ be a covering map between path connected, locally path connected spaces, $x \in X$ and $y = p(x)$. Show that the following are equivalent:

i) The covering is regular, i.e. $p_*(\pi_1(X, x))$ is normal in $\pi_1(Y, y)$.

ii) The action of $\mathrm{Aut}(X/Y)$ on $p^{-1}(y)$ is transitive.

iii) For every closed loop σ at y, if one lifting of σ is closed, then all liftings are closed.

Hand in: during the lecture on Tuesday, December 11th.