Fall term 2018

Prof. D. Kotschick
Dr. J. Stelzig
G. Placini

Topology I

Sheet 7

Exercise 1. Let X be a path-connected, locally path-connected topological group and $\pi : Y \to X$ a path-connected covering space of X. Show that Y admits a topological group structure.

Exercise 2. Let $\pi : Y \to X$ be a n-sheeted covering of a space of X.

a) Show that Y is compact if and only if X is.

b) Prove that Y is Hausdorff if and only if the same is true for X.

Exercise 3. Let X be a simply connected space and $T^n = \underbrace{S^1 \times \cdots \times S^1}_{\text{n times}}$.

a) Show that every map $f : X \to T^n$ is nullhomotopic.

b) Deduce that $\pi_k(T^n) = 0$ for all $k > 1$.

Exercise 4.

a) Consider the sphere S^3 as the space of unit quaternions $\{x \in \mathbb{H} | \|x\| = 1\}$ and identify $\mathbb{R}^3$ with the pure imaginary quaternions $\{a + b\mathbf{i} + c\mathbf{j} + d\mathbf{k} \in \mathbb{H} | a = 0\}$. Prove that the map

$$\begin{aligned} \pi : S^3 \to \quad & SO(3) \\ x \ \mapsto & (p \mapsto xpx^{-1}) \end{aligned}$$

with multiplication in $\mathbb{H}$, is a well defined covering map.

b) Show that $SO(3)$ is homeomorphic to $\mathbb{RP}^3$.

c) Show that on S^3 the group structure induced by $\mathbb{H}$ coincides with the one induced by the covering $\pi : S^3 \to SO(3)$.

Hand in: during the lecture on Tuesday, December 4th.