LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Topology I

Sheet 5

Exercise 1. Prove that $\mathrm{Im}(j) = \ker(\partial)$ in the long homotopy sequence

$$\cdots \xrightarrow{\partial} \pi_n(A, a_0) \xrightarrow{i_*} \pi_n(X, a_0) \xrightarrow{j} \pi_n(X, A, a_0) \xrightarrow{\partial} \pi_{n-1}(A, a_0) \xrightarrow{i_*} \cdots$$

of the pair (X, A).

Exercise 2. Let X, Y be topological spaces and $A \subset X$ a subset.

a) Show that there are short exact sequences

$$0 \to \pi_n(A, a_0) \xrightarrow{i_*} \pi_n(X, a_0) \xrightarrow{j} \pi_n(X, A, a_0) \to 0$$

if there exists a retraction $r : X \to A$.

b) Prove that the sequences above split if $n \geq 3$.

c) Compute $\pi_n(X \times Y, X \times \{y_0\}, (x_0, y_0))$ for $x_0 \in X$ and $y_0 \in Y$.

Exercise 3. Show that the forgetful map $\varphi : \pi_n(X, x_0) \to [S^n, X]$ exhibits $[S^n, X]$ as the set of orbits of the action of $\pi_1(X, x_0)$ on $\pi_n(X, x_0)$.

Exercise 4. Consider the group $\mathrm{SL}(2, \mathbb{R})$ as a subspace of $\mathbb{R}^4$.

a) Prove that, with the topology induced by $\mathbb{R}^4$, $\mathrm{SL}(2, \mathbb{R})$ is a topological group.

b) Show that $\mathrm{SL}(2, \mathbb{R})$ deformation retracts on $\mathrm{SO}(2)$.

c) Compute $\pi_1(\mathrm{SL}(2, \mathbb{R}))$.

Hand in: during the lecture on Tuesday, November 20th.