LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

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Topology I

Sheet 4

Exercise 1. A topological group is a group G which is equipped with a topology such that the multiplication and inverse

$$\begin{aligned} G \times G &\longrightarrow G \\ (g, g') &\longmapsto gg' \end{aligned} \qquad\qquad \begin{aligned} G &\longrightarrow G \\ g &\longmapsto g^{-1} \end{aligned}$$

are continuous maps (where $G \times G$ is equipped with the product topology). Show that for any topological group G, the fundamental group $\pi_1(G)$ is abelian and the action of $\pi_1(G)$ on $\pi_n(G)$ is trivial for all $n \geq 2$.

Exercise 2. Let X be a topological space. For any two points $x_0, x_1 \in X$ and any path γ from x_0 to x_1, we have an isomorphism

$$B_\gamma : \pi_n(X, x_0) \longrightarrow \pi_n(X, x_1).$$

In general, these maps depend on the path γ. What are necessary and sufficient conditions on X to ensure B_γ are independent of γ for all points $x_0, x_1 \in X$?

Exercise 3. Show the fundamental theorem of algebra: Every nonconstant polynomial $p \in \mathbb{C}[x]$ has at least one zero.

Hint: For any nonvanishing polynomial p, $z \mapsto \frac{p(z)}{\|p(z)\|}$ gives rise to a map from S^1 to itself. What is its degree?

Exercise 4. Show that $\pi_1(S^n) = 0$ for $n \geq 2$.

Hint: Show that any homotopy class of maps $S^1 \longrightarrow S^n$ contains a representative which leaves out one point.

Hand in: during the lecture on Tuesday, November 13th.