LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT

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Topology I

Sheet 3

Exercise 1. Show that every homomorphism $\pi_1(S^1, 1) \to \pi_1(S^1, 1)$ can be realized as the induced homomorphism φ_* of a map $\varphi : S^1 \to S^1$.

Exercise 2. Let X and Y be topological spaces, and $x_0 \in X$, $y_0 \in Y$.

a) Show that the map $[(\gamma_X, \gamma_Y)] \mapsto ([\gamma_X], [\gamma_Y])$ defines an isomorphism between $\pi_1(X \times Y, (x_0, y_0))$ and $\pi_1(X, x_0) \times \pi_1(Y, y_0)$.

b) From the above isomorphism it follows that loops in $X \times \{y_0\}$ and $\{x_0\} \times Y$ represent commuting elements of $\pi_1(X \times Y, (x_0, y_0))$. Construct an explicit homotopy demonstrating this.

Exercise 3. Let X be a topological space, $x_0, x_1 \in X$ and $\alpha : I \to X$ a path from x_0 to x_1. Show that the map

$$\begin{aligned} B_\alpha : \pi_1(X, x_1) &\to \pi_1(X, x_0) \\ [\gamma] &\mapsto [\alpha * \gamma * \alpha^{-1}] \end{aligned}$$

where $\alpha^{-1}(t) = \alpha(1-t)$, is a well defined isomorphism.

Exercise 4. Show that there are no retractions $r : X \to A$ in the following cases:

a) $X = \mathbb{R}^3$, with A any subspace homeomorphic to S^1,

b) $X = D^2 \times S^1$ with A its boundary torus $S^1 \times S^1$,

c) $X = D^2 \vee D^2$ with A its boundary $S^1 \vee S^1$,

d) X a disk with two points on its boundary identified and A its boundary $S^1 \vee S^1$,

e) $X = [0,1] \times [0,1]/(1, y) \sim (0, 1-y)$ the Möbius band and A its boundary circle.

Hand in: during the lecture on Tuesday, November 6th.