Fall term 2018

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Topology I

Sheet 1

Exercise 1.

a) Show that the interval $I = [0, 1]$ is connected and path-connected.

b) Prove that the image of a connected (path-connected) space under a continuous map is connected (path-connected).

c) Show that a path connected topological space is connected.

d) Show that $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^n$ for $n \geq 2$.
[Hint: Use point b).]

Exercise 2. Prove that the space $Y = \left\{\left(x, \sin\left(\frac{1}{x}\right)\right) \subset \mathbb{R}^2 | x > 0\right\} \cup \{(0, y) \subset \mathbb{R}^2 | -1 \leq y \leq 1\}$ is connected but not path-connected.

Exercise 3. Let $\|\cdot\|$ be the Euclidean norm on $\mathbb{R}^2$. Consider the function

$$\begin{aligned} d_{\text{SNCF}} : \mathbb{R}^2 \times \mathbb{R}^2 &\to \quad \mathbb{R} \\ (p, q) \quad &\mapsto \begin{cases} \|p - q\| & \text{if } p = \lambda q \text{ for some } \lambda \in \mathbb{R}; \\ \|p\| + \|q\| & \text{otherwise.} \end{cases} \end{aligned}$$

a) Prove that d_{SNCF} defines a metric on $\mathbb{R}^2$.

b) Describe open sets of the topology σ_{SNCF} induced by d_{SNCF}.

c) Is $(\mathbb{R}^2, \sigma_{\text{SNCF}})$ homeomorphic to $\mathbb{R}^2$ endowed with the usual topology induced by the Euclidean metric?

(please turn)

Exercise 4. Let $A, B \subset X$ be closed subsets of a topological space X with $A \cup B = X$. Show that if $f : A \to Y$ and $g : B \to Y$ are continuous maps such that $f_{|A\cap B} = g_{|A\cap B}$ then the map

$$F : X \to Y$$
$$F(x) = \begin{cases} f(x) \text{ if } x \in A \\ g(x) \text{ if } x \in B \end{cases}$$

is continuous.

Hand in: during the lecture on Tuesday, October 23rd.