



LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT



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Topology I

Sheet 14

Exercise 1. Show that the spaces $S^1 \times S^1$ and $S^1 \vee S^1 \vee S^2$ have isomorphic homology groups, but are not homotopy equivalent.

Exercise 2.

- a) For a map $f : S^n \rightarrow S^n$ let $\Sigma(f) : \Sigma(S^n) \cong S^{n+1} \rightarrow S^{n+1}$ be the suspension of f defined by $\Sigma(f)(x, t) = (f(x), t)$. Show that $\deg(\Sigma(f)) = \deg(f)$.
- b) Construct a surjective map $f : S^n \rightarrow S^n$ of degree zero, for each $n \geq 1$.

Exercise 3. Given a map $f : S^{2n} \rightarrow S^{2n}$, show that there is some point $x \in S^{2n}$ with either $f(x) = x$ or $f(x) = -x$. Deduce that every map $\mathbb{R}P^{2n} \rightarrow \mathbb{R}P^{2n}$ has a fixed point.

Exercise 4. Compute the homology of $S^k \times S^l$.

This sheet is not to be handed in; it will be discussed on Monday, February 5th.