Fall term 2017

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Topology I

Sheet 13

Exercise 1. Let Y be a convex set in a Eucledian space. Show that the chain map $i : LC_*(Y) \to C_*(Y)$ given by the inclusion of linear simplices induces an isomorphism in homology.
[Hint: One can define a chain homotopy inverse $q : C_*(Y) \to LC_*(Y)$ by $q(\sigma) = [\sigma(v_0), \dots, \sigma(v_n)]$.]

Exercise 2.

a) Show that if $0 \to A \to B \to C \to 0$ is a short exact sequence of free abelian groups then $B \simeq C \oplus A$.

b) Classify all abelian groups G that fit into the short exact sequence $0 \to \mathbb{Z}_{p^k} \to G \to \mathbb{Z}_{p^k} \to 0$ where $k > 0$ and p is a prime number.
[Fact: Every finite abelian group is isomorphic to a direct sum $\sum \mathbb{Z}_{p_i^{k_i}}$ where p_i's are prime and $k_i > 0$.]

Exercise 3. Show that $H_1(X, A)$ is not isomorphic to $\widetilde{H}_1(X/A)$ if $A = \{\frac{1}{2}; \frac{1}{3}; \dots; \frac{1}{n}; \dots\} \cup \{0\}$ and $X = [0, 1]$.

Exercise 4. Let $f : (X, A) \to (Y, B)$ be a map of pairs. Show that the following diagram commutes:

$$\begin{tikzcd}
\cdots \arrow[r] & H_n(A) \arrow[r, "i_*"] \arrow[d, "f_*"] & H_n(X) \arrow[r, "j_*"] \arrow[d, "f_*"] & H_n(X,A) \arrow[r, "\partial"] \arrow[d, "f_*"] & H_{n-1}(A) \arrow[r] \arrow[d, "f_*"] & \cdots \\
\cdots \arrow[r] & H_n(B) \arrow[r, "i_*"] & H_n(Y) \arrow[r, "j_*"] & H_n(Y,B) \arrow[r, "\partial"] & H_{n-1}(B) \arrow[r] & \cdots
\end{tikzcd}$$

Hand in: during the lecture on Monday, January 29th.