LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT

Fall term 2017

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Topology I

Sheet 12

Exercise 1. Let X be a path connected topological space and $\gamma : (I, \partial I) \to (X, x_0)$ a continuous map. Show that the map $\psi : \pi_1(X, x_0) \to H_1(X)$ given by $[\gamma] \mapsto [\gamma]$ is a homorphism.

Exercise 2. Suppose that a space X deformation retracts onto a subspace $A \subset X$. Show that for any $B \subset A$ there is an isomorphism $H_*(X, B) \simeq H_*(A, B)$.

Exercise 3. Let (X, A) be a topological pair and $i : A \to X$ the inclusion.

a) Describe the map $i_* : H_0(A) \to H_0(X)$ when X is path connected.

b) Compute the group $H_0(X, A)$ and show that the map $H_0(X) \to H_0(X, A)$ is surjective.

c) Let $r : X \to A$ be a retraction. Prove that $i_* : H_n(A) \to H_n(X)$ is injective and that $H_n(X) \simeq H_n(A) \oplus \ker r_*$ for all $n \geq 0$.

Exercise 4. Let X be a topological space and $k > 0$. Prove or disprove the following statements:

a) If $\sigma : \Delta^k \to X$ is a singular cycle of X, then k is odd.

b) If $\sigma + \tau$, with $\sigma, \tau : \Delta^k \to X$, is a singular cycle of X, then k is even.

Hand in: during the lecture on Monday, January 22nd.