Fall term 2017

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Topology I

Sheet 10

Exercise 1. Show that every subgroup H of a free group G is free.

Exercise 2.

a) Show that $f : X \to Y$ is a homotopy equivalence if there exist maps $f, g : Y \to X$ such that $fg \simeq \mathrm{Id}$ and $hf \simeq \mathrm{Id}$.

b) Let $\widetilde{X}$ and $\widetilde{Y}$ be simply-connected covering spaces of the path-connected, locally path-connected spaces X and Y such that $X \simeq Y$. Show that $\widetilde{X}$ is homotopy equivalent to $\widetilde{Y}$.

Exercise 3. Describe the universal covering pr: $\widetilde{X} \to X$ where $X = S^2 \bigcup \{(0,0,z) \in \mathbb{R}^3 | -1 \leq z \leq 1\}$.

Exercise 4. Consider the action of $\mathbb{Z}$ on $X = \mathbb{R}^2 \setminus \{0\}$ defined by $n(x,y) = (2^n x, 2^{-n} y)$. Prove that pr: $X \to X/\mathbb{Z}$ is a covering map but $X/\mathbb{Z}$ is not Hausdorff.

Exercise 5. Consider covering spaces $pr : \widetilde{X} \to X$ with $\widetilde{X}$ connected and X a CW-complex. Show that:

a) Two such covering spaces $pr_1 : \widetilde{X}_1 \to X$ and $pr_2 : \widetilde{X}_2 \to X$ are isomorphic if and only if the restrictions $pr_1 : \widetilde{X}_1^1 \to X^1$ and $pr_2 : \widetilde{X}_2^1 \to X^1$ are isomorphic.

b) $pr : \widetilde{X} \to X$ is a normal covering space if and only if $pr : \widetilde{X}^1 \to X^1$ is normal.

Hand in: during the lecture on Monday, January 8th.