LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Fall term 2017

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Topology I

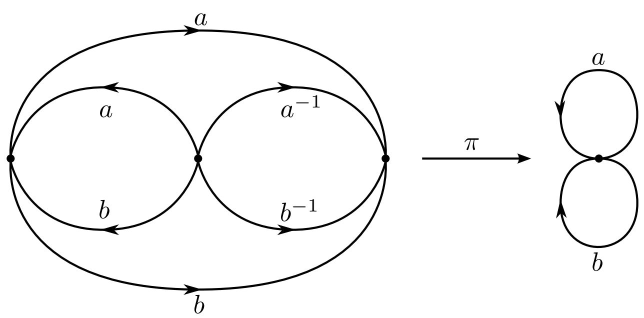
Sheet 8

Exercise 1. Let $X_n = \left\{x \in \mathbb{R}^2 | \parallel x - \left(\frac{1}{n}, 0\right) \parallel = \frac{1}{n}\right\}$ and $X = \bigcup_{n\in\mathbb{N}} X_n$. Show that X is not homotopy equivalent to a CW-complex.

[Hint: Every point of a CW-complex Y has a neighbourhood V such that the inclusion $V \hookrightarrow Y$ is nullhomotopic.]

Exercise 2. Classify CW-complexes with exactly one 0-cell, one 1-cell and one 2-cell up to homotopy equivalence.

Exercise 3. Let $\pi : X \to S^1 \vee S^1$ be the map indicated in the picture below. Determine the subgroup of the fundamental group of $S^1 \vee S^1$ given by the image of the homomorphism $\pi_* : \pi_1(X) \to \pi_1(S^1 \vee S^1)$ induced by π.



Exercise 4. Let X be a CW-complex such that $\pi_n(X) = 0$ for all $n \geq 0$.

a) Let $F = \text{Id} : X \to X$ and $p_0 \in X$ a 0-cell. Show that F is homotopic to a map F_0 such that $F_0(X^0) = p_0$.

b) Show that F_0 is homotopic (rel. X^0) to a map F_1 such that $F_1(X^1) = p_0$.

c) Use induction as in the proof of the cellular approximation theorem to prove that X is contractible.

Hand in: during the lecture on Monday, December 11th.