LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Fall term 2017

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Topology I

Sheet 7

Exercise 1. Let (X, A) be a pair that satisfies the homotopy extension property. Suppose that, for a given space Y, for every map $f : A \to Y$ there exists an extension $\tilde{f} : X \to Y$. Prove that $g : A \to Z$ extends to a map $\tilde{g} : X \to Z$ if Z is homotopy equivalent to Y.

Exercise 2. Let X be any topological space and $\pi_n(X, x_0)$ be the n-th homotopy group of X with basepoint $x_0 \in X$. A path $\gamma : I \to X$ from $x_0 = \gamma(0)$ to another basepoint $x_1 = \gamma(1)$ associates to each map $f : (I^n, \partial I^n) \to (X, x_1)$ a new map $\gamma f : (I^n, \partial I^n) \to (X, x_0)$ that can be defined as

$$\gamma f(y_1, \ldots, y_n) = \begin{cases} f(2y_1 - \frac{1}{2}, \ldots, 2y_n - \frac{1}{2}) & \text{for } \max\{|y_i - \frac{1}{2}|\} \leq \frac{1}{4} \\ \gamma(4 \cdot \mathrm{d}(y, \partial I^n)) & \text{for } \max\{|y_i - \frac{1}{2}|\} \geq \frac{1}{4} \end{cases}$$

Prove the following property for $f, g : (I^n, \partial I^n) \to (X, x_1)$ and $\gamma : I \to X$ as above

$$\gamma(f + g) \simeq \gamma f + \gamma g.$$

Exercise 3. A **topological group** is a topological space with a group structure such that the multiplication $(x, y) \mapsto x \cdot y$ and the inverse $x \mapsto x^{-1}$ are continuous maps.

a) Prove that any subgroup H of a topological group G is again a topological group.

b) Show that $GL(n, \mathbb{R})$, $O(n)$, $SO(n)$ and $SL(n, \mathbb{R})$ are topological groups.

c) Prove that $\pi_1(G, e)$ is abelian if G is a topological group and $e \in G$ the identity element.

Exercise 4. Assume $\mathrm{pr} : Y \to X$ is a n-sheeted covering of a compact space X.

a) Show that Y is compact.

b) Prove that X is Hausdorff iff the same is true for Y.

Hand in: during the lecture on Monday, December 4th.