Fall term 2017

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Topology I

Sheet 6

Exercise 1. Consider connected countable one dimensional CW-complexes (i.e. countably many cells).

a) Use Zorn's lemma to find a maximal tree (a tree is a CW-subcomplex which becomes disconnected whenever one removes a vertex).

b) Use Zorn's lemma to show that this tree is contractible.
[Consider contractible subtrees]

c) Show that a connected 1-dimensional CW complex is homotopy equivalent to a CW complex with exactly one zero cell.

Exercise 2. Let $f, g : A \longrightarrow Y$ be attaching maps for (X, A) and assume that there is a homeomorphism $\varphi : Y \longrightarrow Y$ such that $\varphi \circ f = g$. Show that $Y \cup_f X$ and $Y \cup_g X$ are homeomorphic.

Exercise 3. Let X be a CW-complex with $x_0 \in X$ a 0-cell and Y a connected CW-complex with no 1-cells (therefore Y has only one 0-cell y_0). Show that if $f, g : X \to Y$, with $f(x_0) = g(x_0) = y_0$, are homotopic maps then f is homotopic to g relative to x_0.

Exercise 4. Let $X = S^n \times \cdots \times S^n$ be the product of $k \geq 2$ spheres of dimension $n \geq 1$.

a) Show that X is a CW-complex which only has cells in dimension $0, n, 2n, \ldots, kn$.

b) Prove that the n-skeleton X^n of X is the one point union of k spheres $S^n \vee \cdots \vee S^n$.

c) Show that every map $f : S^n \to X$ is homotopic to a map g such that $g(S^n) \subset X^n$ and $g(e^0) = x_0$ (where e^0 is a fixed point in S^n and x_0 is the common point of the spheres in the one point union).

d) Suppose $n \geq 2$ and let $g_1, g_2 : S^n \to X^n$ be maps such that $g_1(e^0) = g_2(e^0) = x_0$ and $i \circ g_1 \simeq i \circ g_2$ (here $i : X^n \to X$ is the inclusion). Prove that g_1 is homotopic to g_2 relative to e^0.

Hand in: during the lecture on Monday, November 27th.