Fall term 2017

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Topology I

Sheet 4

Exercise 1. Let $f, g : S^1 \to S^1$ be continuous maps. Prove that:

a) $\deg(f \circ g) = \deg(f) \cdot \deg(g)$.
[Hint: Use the lemmas from the lecture.]

b) If f is a homeomorphism then $\deg(f) \in \{+1; -1\}$.

Exercise 2.

a) Show that every (nontrivial) CW-complex has at least one 0-cell.

b) Prove that a CW-complex X is path-connected if its 1-skeleton X^1 is path-connected.

Exercise 3. Let S^∞ be the CW-complex defined as $S^\infty = \{x \in \mathbb{R}^\infty | \sum_{n=1}^{\infty} x_n^2 = 1\}$.

a) Show that the map

$$\begin{array}{cccc} h_t : & S^\infty & \longrightarrow & S^\infty \\ & (x_1, x_2, \ldots) & \mapsto & \dfrac{((1-t)x_1, tx_1 + (1-t)x_2, tx_2 + (1-t)x_3, \ldots)}{N} \end{array}$$

where N is the norm of the point at the numerator, is a well defined homotopy.

b) Show that the map

$$\begin{array}{cccc} h_t : & A & \longrightarrow & S^\infty \\ & (0, x_2, x_3, \ldots) & \mapsto & \dfrac{(t, (1-t)x_2, (1-t)x_3, \ldots)}{N} \end{array}$$

where $A = \{x \in S^\infty | x_1 = 0\}$, defines a homotopy and conclude that S^∞ is contractible.

Exercise 4. Use the Gram-Schmidt process to show that $GL(n, \mathbb{R})$ is homotopy equivalent to $O(n)$.

(please turn)

Exercise 5. Let $a, b \in \mathbb{Z}$ and consider the map $f_{a,b} : S^1 \to T^2 = S^1 \times S^1$ defined by $z \mapsto (z^a, z^b)$. When are two maps $f_{a,b}$ and $f_{c,d}$ homotopic?

Hand in: during the lecture on Monday, November 13th.