LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Fall term 2017

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Topology I

Sheet 1

Exercise 1. A topological space X is **path connected** if for any two points $x, y \in X$ there exists a continuous path $\gamma : [0,1] \to X$ such that $\gamma(0) = x$ and $\gamma(1) = y$.

1. Show that a path connected topological space is connected.
2. Prove that the space $Y = \left\{\left(x, \sin\left(\frac{1}{x}\right)\right) \subset \mathbb{R}^2 | x > 0\right\} \cup \{(0, y) \subset \mathbb{R}^2 | -1 \leq y \leq 1\}$ is connected but not path connected.
3. Show that the image of a connected space under a continuous map is connected.

Exercise 2. Show that a map $f : X \to Y$ is continuous if and only if the preimage of any closed set is closed.

Exercise 3. Give an explicit homeomorphism between the following pairs of spaces:

1. $\mathbb{R}^n$ and $S^n \setminus \{p\}$ where p is any point on the sphere S^n.
2. $D^n \times D^m$ and D^{n+m} where $D^n = \{x \in \mathbb{R}^n | \parallel x \parallel \leq 1\}$.
3. S^1 and the quotient $I/\backsim$ of $I = [0,1] \subset \mathbb{R}$ by the relation $0 \backsim 1$.

Exercise 4. Let $A, B \subset X$ be closed subsets of a topological space X with $A \cup B = X$. Show that if $f : A \to Y$ and $g : B \to Y$ are continuous maps such that $f_{|A \cap B} = g_{|A \cap B}$ then the map

$$F : X \to Y$$
$$F(x) = \begin{cases} f(x) \text{ if } x \in A \\ g(x) \text{ if } x \in B \end{cases}$$

is continuous.

Hand in: during the lecture on Monday, October 23rd.