Summer term 2020

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Mathematical Gauge Theory II

Sheet 8

Exercise 1. (Almost complex manifolds I) Show that a closed connected almost complex manifold X admits a canonical Spin^c structure $\mathfrak{s}$.

Exercise 2. (Almost complex manifolds II) Let $\mathfrak{s}_L$ be the canonical Spin^c structure twisted by a line bundle L on a closed connected almost complex manifold X. Show that for generic parameters the expected dimension of the moduli space of irreducible solutions is

$$\dim(\mathcal{M}^*_\omega) = c_2((V_L)_+) = c_1^2(L) + c_1(L)c_1(X)$$

where $c_1(X)$ is the first Chern class of the complex vector bundle TX.

Exercise 3. (Reducible solutions II) Let (X, g) be a smooth closed oriented Riemannian 4-manifold endowed with a Spin^c-structure $\mathfrak{s}$. Show that if $b_2^+(X) > 0$ and $c_1(L_\mathfrak{s}) \neq 0$, then for a generic metric on X the unperturbed Seiberg-Witten equations do not admit reducible solutions.

Exercise 4. (Theorem 5.16) Show that the maps

$$\mathfrak{e}\colon\ \mathcal{G}^\perp \times S \longrightarrow \mathcal{C}_\mathfrak{s}$$
$$(e^{if}, (A_0 + a, \Phi)) \longmapsto (A_0 + a - i\,\mathrm{d}\,f, e^{if}\Phi)$$

and

$$\mathfrak{e}^{-1}\colon\ \mathcal{C}_\mathfrak{s} \longrightarrow \mathcal{G}^\perp \times S$$
$$(A_0 + b, \Psi) \longmapsto (e^{-G(\mathrm{d}^*b)}, A_0 + b - \mathrm{d}(G(\mathrm{d}^*b)), e^{G(\mathrm{d}^*b)}\Psi))$$

defined in Theorem 5.16 in the notes are inverse to each other.

You can email the solutions until Tuesday, June 23rd at noon.