Topology II

Sheet 11

Exercise 1. Show that $\mathbb{CP}^2 \sharp \overline{\mathbb{CP}^2}$ and $\mathbb{CP}^2 \sharp \mathbb{CP}^2$ do not dominate each other.

Exercise 2.

a) Show that $cl(X_1 \times \cdots \times X_k) = \sum_{i=1}^{k} cl(X_i)$.

b) Conclude that if $M \geq X_1 \times \cdots \times X_k$ then $cl(M) \geq \sum_{i=1}^{k} cl(X_i)$.

Exercise 3. Show that every map $\mathbb{CP}^n \longrightarrow \mathbb{CP}^n$ has degree d^n for some $d \in \mathbb{Z}$. Conversely, prove that any integer d^n can be realised as the degree of a map $\mathbb{CP}^n \longrightarrow \mathbb{CP}^n$.

Exercise 4. Show that the 4-dimensional torus T^4 does not dominate the connected sum $\sharp_k(S^1 \times S^3)$ of $k \geq 2$ copies of $S^1 \times S^3$.

Exercise 5. Prove that for all $k \geq 1$ there exists a finite covering space X of $T^{2n} \sharp \mathbb{CP}^n$ and a map $f \colon X \longrightarrow \underbrace{\mathbb{CP}^n \sharp \cdots \sharp \mathbb{CP}^n}_{k \text{ times}}$ with $deg(f) = 1$.

Hand in: during the exercise class on Monday, July 22nd.