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MATHEMATISCHES INSTITUT



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# Topology II

Sheet 10

**Exercise 1.** Prove that the map  $T_X: \Omega_n(X) \rightarrow H_n(X)$  is a well-defined homomorphism.

**Exercise 2.** Let  $M_g$  be the closed orientable surface of genus  $g$ . Show that if a map  $f: M_g \rightarrow M_h$  of nonzero degree exists then  $g \geq h$ . Conversely show that if  $g \geq h$  then there exists a map  $f: M_g \rightarrow M_h$  of degree 1.

**Exercise 3.** Given two compact connected oriented manifolds  $M$  and  $N$  of dimension  $4n$ , show that if  $M \geq N$  then  $b_{2n}(M) - |\sigma(M)| \geq b_{2n}(N) - |\sigma(N)|$ .

**Exercise 4.** Let  $M$  be a compact orientable  $4k$ -dimensional manifold with  $\partial M \neq \emptyset$ . Prove that  $\sigma(D(M)) = 0$  where  $D(M) = M \cup_{\partial M} \overline{M}$  is the double of  $M$ .

Hand in: during the exercise class on Monday, July 15th.