Summer term 2019

Prof. D. Kotschick
G. Placini

Topology II

Sheet 9

Exercise 1. Let $Q\colon V \times V \longrightarrow \mathbb{R}$ be a symmetric bilinear non-degenerate form on an $\mathbb{R}$-vector space V.

a) Suppose Q has p positive and q negative eigenvalues. Show that for any Q-isotropic subspace $L \subset V$

$$\dim L \leq \min\{p, q\}.$$

b) Show that $\sigma(Q) = 0$ if and only if there exists $L \subset V$ such that $Q_{|L\times L} \equiv 0$ and $\dim V = 2 \dim L$.

Exercise 2. Show that the signature $\sigma(M)$ is congruent $\mod 2$ to the Euler characteristic $\chi(M)$ for any compact connected oriented manifold M.

Exercise 3.

(a) Show that there exist a continuous surjective map $\pi : \mathbb{C}\mathrm{P}^{2m+1} \to \mathbb{H}\mathrm{P}^m$ such that $\pi^{-1}(x)$ is homeomorphic to $\mathbb{C}\mathrm{P}^1$ for all $x \in \mathbb{H}\mathrm{P}^m$.

(b) Use point (a) to conclude that the odd dimensional complex projective space $\mathbb{C}\mathrm{P}^{2m+1}$ is bordant to the empty manifold.

Exercise 4. Prove that the map $T_X\colon \Omega_1(X) \longrightarrow H_1(X)$ is surjective.

Hand in: during the exercise class on Monday, July 8th.