



Summer term 2019

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# Topology II

Sheet 8

**Exercise 1.** Show that the Betti number  $b_{2n+1}(M)$  is even for a compact connected orientable manifold  $M$  of dimension  $4n + 2$ .

**Exercise 2.** Let  $M$  be a closed oriented 3-manifold. Assume that  $H^1(M; \mathbb{R}) \neq 0$  is even-dimensional. Prove that for every  $0 \neq \alpha \in H^1(M; \mathbb{R})$  there is a  $\beta \in H^1(M; \mathbb{R})$ , linearly independent of  $\alpha$ , such that  $\alpha \smile \beta = 0$  in  $H^2(M; \mathbb{R})$ .

**Exercise 3.** Prove that there exists no orientation reversing homotopy equivalence  $h : \mathbb{C}P^{2n} \rightarrow \mathbb{C}P^{2n}$  (without making use of the signature).

**Exercise 4.**

Show that if  $M$  is a compact contractible  $n$ -manifold then its boundary  $\partial M$  is a homology  $(n - 1)$ -sphere, that is,  $H_i(\partial M; \mathbb{Z}) \simeq H_i(S^{n-1}; \mathbb{Z})$  for all  $i$ .

Hand in: during the exercise class on Monday, July 1st.