Summer term 2019

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# Topology II

Sheet 7

**Exercise 1.** Let $G$ be a (discrete) group and $M$ a manifold. Recall that an action of $G$ on $M$ is called a covering space action if the following condition is satisfied: For any $p \in M$, there exists a neighbourhood $U \subset M$ of $p$, s.t. for all $g \in G$, $gU \cap U \neq \emptyset$ implies $g = e_G$.
Show that the quotient $M/G$ is a manifold if $G$ acts by a covering action by homeomorphisms on $M$ that satisfies the following condition

- For any $p, q \in M$ there exist open neighbourhoods $U \ni p$ and $V \ni q$ such that $g \cdot U \cap V$ is non-empty only for finitely many $g \in G$.

**Exercise 2.** Show that $H^0_c(X; G) = 0$ if $X$ is path-connected and noncompact.

**Exercise 3.** Show that $H^n_c(X \times \mathbb{R}; G) \simeq H^{n-1}_c(X; G)$ for all $n$.

**Exercise 4.**

a) Use the Universal Coefficient Theorem to show that if $H_*(X; \mathbb{Z})$ is finitely generated, so the Euler characteristic $\chi(X) = \sum_n (-1)^n \text{rank} H_n(X; \mathbb{Z})$ is defined, then we have $\chi(X) = \sum^n (-1)^n \dim H^n(X; F)$ with coefficient field $F = \mathbb{Q}, \mathbb{R}, \mathbb{C}$ or $\mathbb{Z}_p$ for $p$ prime.

b) Show that the Euler characteristic $\chi(X)$ of a compact odd dimensional manifold $M$ vanishes.

Hand in: during the exercise class on Monday, June 24th.