LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT

Summer term 2019

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Topology II

Sheet 6

Exercise 1. Show that $M \times N$ is orientable if and only if M and N are both orientable.

Exercise 2. Show that deleting a point from a manifold does not affect orientability of the manifold.

Exercise 3. Show that every compact oriented connected n-dimensional manifold M admits a degree one map to S^n.

Exercise 4. Let M be a path-connected n-dimensional manifold. Let $c : [0,1] \to M$ be a curve. We define an orientation along c to be an assignment of a local orientation μ_t (i.e. a generator of $H_n(M, M \setminus \{c(t)\})$) to each $t \in [0,1]$, satisfying the following compatibility condition: for every $t \in [0,1]$ there exists an open interval U around t and a ball $B \subset M$ around $c(t)$ with $c(U) \subset B$ such that the local orientations on $c(s)$ for $s \in U$ are given by the image of a generator of $H_n(M, M \setminus B)$.

Let $h : \pi_1(M,p) \to \mathbb{Z}_2$ be the map defined as follows. Set $h(\alpha) = 0$ if for a curve $c : [0,1] \to M$ representing $\alpha \in \pi_1(M,p)$ we have $\mu_0 = \mu_1 \in H_n(M, M \setminus \{p\})$, and $h(\alpha) = 1$ otherwise.

1. Prove that h is a well defined homomorphism.
2. Prove that M is orientable if and only if h is trivial.
3. Prove that if $H_1(M) = 0$, then M is orientable.

Hand in: during the lecture on Thursday, June 13th.