LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Summer term 2019

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Topology II

Sheet 4

Exercise 1.

Let X be a topological space and R a commutative ring.

a) Show that $2 \cdot \alpha \smile \alpha = 0 \in H^{2k}(X;R)$ for all $\alpha \in H^k(X;R)$ with k odd.

b) Prove that $H^*(X;R)$ is a commutative ring if $H^k(X;R) = 0$ whenever k is odd.

Exercise 2. Let X and Y be topological spaces with $x \in X$ and $y \in Y$ such that x (respectively y) has a contractible neighbourhood $U \subset X$ (resp. $V \subset Y$). Show that

$$\alpha \smile \beta = 0 \in H^{k+l}(X \vee Y;G)$$

for $\alpha \in H^k(X;G)$, $\beta \in H^l(Y;G)$ and $k,l > 0$ where we use the isomorphism $H^i(X \vee Y;G) \simeq H^i(X;G) \oplus H^i(Y;G)$ when $i > 0$.

Exercise 3.

a) Determine the cup product $\smile : H^1(T^2;\mathbb{Q}) \times H^1(T^2;\mathbb{Q}) \longrightarrow H^2(T^2;\mathbb{Q})$.

b) Prove that every map $f : S^2 \longrightarrow T^2$ induces the trivial map on cohomology groups of positive degree with coefficients in $\mathbb{Q}$.

Exercise 4. Compute the cup product structure in $H^*(\Sigma_g;\mathbb{Q})$ for Σ_g the closed orientable surface of genus g by using the quotient map from Σ_g to the one point union of g tori.

Hand in: during the lecture on Thursday, May 23rd.