Summer term 2019

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Topology II

Sheet 3

Exercise 1.

Let (X, x_0), (Y, y_0) be pointed topological spaces such that x_0 (respectively y_0) has a contractible neighbourhood $U \subset X$ (resp. $V \subset Y$). Given an Abelian group G, compute the cohomology groups $H^n(X \vee Y; G)$ of the wedge sum $X \vee Y$.

Exercise 2. Let X be a Moore space $M(\mathbb{Z}_m, n)$ obtained by attaching a cell e^{n+1} to S^n by a map of degree m.

1. Show that the quotient map $X \to X/S^n = S^{n+1}$ induces the trivial map on $\tilde{H}_i(-;\mathbb{Z})$ for all i, but not on $H^{n+1}(-;\mathbb{Z})$. Deduce that the splitting in the universal coefficient theorem for cohomology cannot be natural.
2. Show that the inclusion $S^n \hookrightarrow X$ induces the trivial map on $\tilde{H}^i(-;\mathbb{Z})$ for all i, but not on $H_n(-;\mathbb{Z})$.

Exercise 3. Compute the cohomology groups of $\mathbb{R}\mathrm{P}^n$ with coefficients in $\mathbb{Z}_2$, $\mathbb{Q}$ using cellular cohomology.

Exercise 4. Let X be a finite CW complex and $H^i(X^i, X^{i-1}; \mathbb{Q})$ its i-th cellular cochain group. Show that

$$\sum_{i \in \mathbb{Z}} (-1)^i \dim H^i(X^i, X^{i-1}; \mathbb{Q}) = \sum_{i \in \mathbb{Z}} (-1)^i \dim H^i(X; \mathbb{Q}) \, .$$

Hand in: during the lecture on Thursday, May 16th.