Topology II

Sheet 2

Exercise 1.

1. Compute the cohomology groups $H^i(S^n; G)$ for $i, n \geq 0$ and G an arbitrary Abelian group.

2. Show that if $f : S^n \to S^n$ has degree d then $f^* : H^n(S^n; G) \to H^n(S^n; G)$ is multiplication by d.

Exercise 2.

a) Let Σ_g be the closed orientable surface of genus g. Compute the cohomology groups $H^i(\Sigma_g; G)$ for all i and n with arbitrary coefficients.

b) Compute the cohomology groups $H^i(\mathbb{CP}^n; G)$ for all i and n with arbitrary coefficients.

Exercise 3. Compute the cohomology groups $H^i(\mathbb{RP}^n; G)$ for all i and n with arbitrary coefficients.

Exercise 4. Compute the cohomology groups of $S^n \times S^m$ with arbitrary coefficients for all $n, m \in \mathbb{N}$.

Remark. You are allowed to assume the homology of all spaces in this Sheet.

Hand in: during the lecture on Thursday, May 9th.