Summer term 2019

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Topology II

Sheet 1

Exercise 1. Show that $\text{Ext}(H, G)$ is a contravariant functor of H for fixed G, and a covariant functor of G for fixed H.

Exercise 2. Prove the following statements:

1. Every short exact sequence $0 \to A \to B \to C \to 0$ of free Abelian groups splits.
2. $\text{Ext}(H_1 \oplus H_2, G) = \text{Ext}(H_1, G) \oplus \text{Ext}(H_2, G)$ for arbitrary Abelian groups H_1, H_2 and G.

Exercise 3. Show that the maps $G \xrightarrow{n} G$ and $H \xrightarrow{n} H$ multiplying each element by the integer n induce multiplication by n in $\text{Ext}(H, G)$.

Exercise 4.

1. Compute $\text{Hom}(H, G)$ and $\text{Ext}(H, G)$ for $H = \mathbb{Z}, \mathbb{Z}_n$ and $G = \mathbb{Z}, \mathbb{Z}_n$.
2. Let X be a topological space with finitely generated homology groups $H_i(X, \mathbb{Z})$. Deduce from the first part that $H^i(X, \mathbb{Z}) \cong F_i \oplus T_{i-1}$ where T_i is the torsion subgroup of $H_i(X, \mathbb{Z})$, $F_i = H_i(X, \mathbb{Z})/T_i$ and $H^i(X, \mathbb{Z})$ is the i-th cohomology group of X with integer coefficients.

Hand in: during the lecture on Thursday, May 2nd.