Summer term 2018

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Topology II

Sheet 10

Exercise 1. Show that if M is a compact contractible n-manifold then its boundary ∂M is a homology $(n-1)$-sphere, that is, $H_i(\partial M;\mathbb{Z}) \simeq H_i(S^{n-1};\mathbb{Z})$ for all i.

Exercise 2. For a space X, let X^+ be the one-point compactification. If the added point, denoted ∞, has a neighborhood in X^+ that is a cone with ∞ the cone point, show that the evident map $H^n_c(X;G) \to H^n(X^+,\infty;G)$ is an isomorphism for all n.
[Question: Does $X = \mathbb{Z}\times\mathbb{R}$ satisfy the hypothesis?]

Exercise 3. Let M be a 7-dimensional topological manifold with

$$H_7(M;\mathbb{Z}) = \mathbb{Z} \quad H_6(M;\mathbb{Z}) = \mathbb{Z} \quad H_5(M;\mathbb{Z}) = \mathbb{Z}_2 \quad H_4(M;\mathbb{Z}) = \mathbb{Z}\oplus\mathbb{Z}_3$$

and denote by T_3 the torsion subgroup of $H_3(M;\mathbb{Z})$.
Compute the homology and cohomology groups of M up to torsion in terms of T_3.

Exercise 4. In the setting of the previous exercise, discuss how the ring structure of $H^*(M;\mathbb{Z})$ depends on T_3.

Hand in: during the lecture on Monday, July 2nd.