LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Summer term 2018

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Topology II

Sheet 9

Exercise 1. Let $\lambda : [0,1] \to \mathbb{R}\mathrm{P}^2$ represent the generator of $H_1(\mathbb{R}\mathrm{P}^2)$ and f represent the nontrivial element $\alpha \in H^1(\mathbb{R}\mathrm{P}^2; \mathbb{Z}_2)$. Then $f(\lambda) = 1$.

a) Let σ be a singular 2-simplex that maps edges $(0,1)$ and $(1,2)$ along λ and $(0,2)$ by a constant map. Compute $(f \smile f)(\sigma)$ and $(f \smile f)(c)$ where c is the constant 2-simplex.

b) Use the previous point to prove by contradiction that $f \smile f$ is not a coboundary.

c) Deduce that $H^*(\mathbb{R}\mathrm{P}^2; \mathbb{Z}_2) = \mathbb{Z}_2[\alpha]/\alpha^3$.

Exercise 2. Suppose $X = U \cup V$ with U, V open sets such that $\tilde{H}_*(U) = \tilde{H}_*(V) = 0$. Show that $\alpha \smile \beta = 0$ for all cohomology classes $\alpha, \beta \in H^*(X)$ of positive degree.

Exercise 3. Compute the cohomology ring of the suspension $\Sigma(X)$ in terms of the cohomology of X.

Exercise 4. Let M, N be manifolds. Compute the cohomology ring of the one point union $M \vee N$ in terms of those of M and N.

Exercise 5. Prove that $M \times N$ is orientable if and only if M and N are.

Exercise 6. Let M be a compact connected orientable manifold of dimension $4n + 2$.

a) Show that the Betti number $b_{2n+1}(M)$ is even.

b) Show that the Euler characteristic $\chi(M)$ is even.

[Requires Poincaré duality.]

Hand in: during the lecture on Monday, June 25th.