LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Summer term 2018

Prof. T. Vogel
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Topology II

Sheet 7

Exercise 1. Compute the cohomology of the Klein bottle and the real projective space $\mathbb{R}\mathrm{P}^n$ with coefficients in $\mathbb{Z}, \mathbb{Z}_2$ and $\mathbb{Q}$.

Exercise 2. Let $(X, Y), (A, C), (B, D)$ be pairs of spaces such that $X = \mathring{A} \cup \mathring{B}$ and $Y = \mathring{C} \cup \mathring{D}$ (in the relative topology). Prove the existence of the Mayer-Vietoris sequence in cohomology in the relative case:

$$\cdots \longrightarrow H^n(A \cap B, C \cap D) \longrightarrow H^n(A, C) \oplus H^n(B, D) \longrightarrow H^n(X, Y) \longrightarrow \cdots .$$

Exercise 3. Compute the cohomology groups $H^*(\mathbb{R}\mathrm{P}^2, \mathbb{R}\mathrm{P}^1; \mathbb{Z}_2)$.

Hand in: during the lecture on Tuesday, June 5th.