Summer term 2018

Prof. T. Vogel
G. Placini

Topology II

Sheet 5

Exercise 1. Show that

$$\begin{aligned} C \otimes C' &\longrightarrow C' \otimes C \\ a \otimes b &\mapsto (-1)^{|a||b|} b \otimes a \end{aligned}$$

is a chain map.

Exercise 2.

a) Let $P : C_*(X) \otimes C_*(Y) \longrightarrow C_*(X \times Y)$ be an Eilenberg-Zilber equivalence and $A \subset X, B \subset Y$. Show that P maps $C_*(A) \otimes C_*(Y), C_*(X) \otimes C_*(B)$ to the images of $C_*(A \times Y), C_*(X \times B)$ in $C_*(X \times Y)$ under the inclusion mappings.

b) Conclude that if $c \in C_*(X)$ is a cycle relative to A and $d \in C_*(Y)$ is a cycle relative to B, then $P(c \otimes d)$ is a cycle relative to $A \times Y \cup X \times B$.

Exercise 3. In the setting of 2). Let $P' : C_*(X) \otimes C_*(Y) \longrightarrow C_*(X \times Y)$ be another Eilenberg Zilber equivalence. Show that $P'(c \otimes d)$ and $P(c \otimes d)$ are homologous relative to $A \times Y \cup X \times B$.

Exercise 4. Let Q^{Δ}, P be the maps defined in the lecture. Prove that the map

$$\begin{aligned} Q : C_*(X \times Y) \to \quad & C_*(X) \otimes C_*(Y) \\ \sigma \quad & \mapsto (pr_X \sigma)_* \otimes (pr_Y \sigma)_* \left(Q^{\Delta}(d_n)\right) \end{aligned}$$

defines a natural chain homotopy inverse of P.

Hand in: during the lecture on Monday, May 14th.