Summer term 2018

Prof. T. Vogel
G. Placini

Topology II

Sheet 3

Exercise 1. Let X be a Moore space $M(\mathbb{Z}_m, n)$ obtained by attaching a cell e^{n+1} to S^n by a map of degree m. Show that the quotient map $X \to X/S^n = S^{n+1}$ induces the trivial map on $\tilde{H}_i(-;\mathbb{Z})$ for all i, but not on $H_i(-;\mathbb{Z}_m)$ for all i. Deduce that the splitting in the universal coefficient theorem cannot be natural.

Exercise 2. Use the universal coefficient theorem to show that if $H_*(X;\mathbb{Z})$ is finitely generated, so the Euler characteristic $\chi(X) = \sum_n (-1)^n \mathrm{rank} H_n(X;\mathbb{Z})$ is defined, then for any prime p we have $\chi(X) = \sum_n (-1)^n \mathrm{rank} H_n(X;\mathbb{Z}_p)$.

Exercise 3. Show that $\mathrm{Tor}(A, \mathbb{Q}/\mathbb{Z})$ is isomorphic to the torsion subgroup of A. Deduce that A is torsionfree if and only if $\mathrm{Tor}(A, B) = 0$ for all B.

Exercise 4. Compute the homology of $\mathbb{R}\mathrm{P}^\infty$ with coefficients in $\mathbb{Z}_2, \mathbb{Q}$.

Hand in: during the lecture on Monday, April 30th.