LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT

Summer term 2018

Prof. T. Vogel
G. Placini

Topology II

Sheet 2

Exercise 1. Compute the homology of the Klein bottle with coefficients in $\mathbb{Z}, \mathbb{Z}_2$ and $\mathbb{Q}$.
[Hint: use CW-structure.]

Exercise 2. Assume that the rows in the following commutative diagram of Abelian groups are exact. Show that if f_2, f_4 are isomorphism while f_1 is surjective and f_5 is injective, then f_3 is an isomorphism.

$$\begin{array}{ccccccccc} A_1 & \longrightarrow & A_2 & \longrightarrow & A_3 & \longrightarrow & A_4 & \longrightarrow & A_5 \\ \downarrow f_1 & & \downarrow f_2 & & \downarrow f_3 & & \downarrow f_4 & & \downarrow f_5 \\ B_1 & \longrightarrow & B_2 & \longrightarrow & B_3 & \longrightarrow & B_4 & \longrightarrow & B_5 \end{array}$$

Exercise 3. Show that $Z_m \otimes Z_n \cong \mathbb{Z}_d$ if $gcd(m,n) = d$.

Exercise 4. Let X be a path connected space. Show that there exists an isomorphism

$$\pi_1(X)_{ab} \otimes G \longrightarrow H_1(X; G)$$

where G is an Abelian group and $\pi_1(X)_{ab}$ the abelianization of $\pi_1(X)$.

Exercise 5. Let G be a group (not necessarily Abelian) and denote by $\mathrm{Aut}(G)$ the group of isomorphisms $\varphi : G \to G$.
Show that $G = \mathbb{Z}_2$ if and only if $\mathrm{Aut}(G) = \{1\}$.

Hand in: during the lecture on Monday, April 23rd.