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Geometric Group Theory

Sheet 10

Exercise 1. Consider the matrices

$$a := \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad b := \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

in $\mathrm{GL}(2, \mathbb{Q})$ and the subgroup $G := \langle a, b \rangle$ of $\mathrm{GL}(2, \mathbb{Q})$. Prove that G has exponential growth.

Exercise 2. Let G be a finitely generated group with the following properties:

- The group G is not of exponential growth.
- G is quasi-isometric to $G \times G$.

Show that this implies that G is of intermediate growth.

Exercise 3. Prove that the fundamental group $\pi_1(\Sigma_g)$ of a closed oriented surface Σ_g of genus $g \geq 2$ admits a surjective homomorphism to a free group on two generators. Conclude that $\pi_1(\Sigma_g)$ has exponential growth without using the Riemannian volume growth.

Exercise 4. Prove that the Thompson's group F has exponential growth.

You can hand in your solutions during the exercise classes.