LUDWIG-
MAXIMILIANS-
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MATHEMATISCHES INSTITUT

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Geometric Group Theory

Sheet 8

Exercise 1. For each of the following group actions name one of the conditions of the Schwarz-Milnor lemma that is satisfied, and one that is not.

a) The action of $\mathrm{SL}(2,\mathbb{Z})$ on $\mathbb{R}^2$ given by matrix multiplication.

b) The action of $\mathbb{Z}$ on $X := \{(r^3, s) | r, s \in \mathbb{Z}\}$ (with respect to the metric induced from the Euclidean metric on $\mathbb{R}^2$) that is given by

$$\begin{aligned} \mathbb{Z} \times X &\to X \\ (n, (r^3, s)) &\mapsto (r^3, s+n) \end{aligned}$$

Exercise 2. Characterise all group homomorphisms between finitely generated groups that are quasi-isometries.

Exercise 3.

a) Is the property of being generated by 2018 elements invariant under quasi-isometries for finitely generated groups?

b) Is the property of being isomorphic to a subgroup of $\mathbb{Z}^{2018}$ invariant under quasi-isometries for finitely generated groups?

c) Is the property of being infinite and torsion-free invariant under quasi-isometries for finitely generated groups?

d) Is the property of being a 2018-torsion group invariant under quasi-isometries for finitely generated groups?

e) Is the property of being a free product of two non-trivial groups invariant under quasi-isometries for finitely generated groups?

(please turn)

Exercise 4. Let G be a finitely generated group with finite generating set $S \subset G$ and let $H \subset G' \subset G$ be subgroups. Moreover, let $H \subset G'$ be quasi-dense with respect to d_S, i.e., there exists a $c \in \mathbb{R}_{>0}$ with

$$\forall g \in G' \quad \exists h \in H \quad d_S(g, h) \leq c.$$

Prove that then H has finite index in G'.

Exercise 5. Let X, X' and Y be metric spaces and suppose that X and X' are quasi-isometric.

a) Are $X \times Y$ and $X' \times Y$ then also quasi-isometric?

b) Does the same answer hold when X, X' and Y are finitely generated groups and X and X' are quasi-isometric?

Exercise 6.

a) Let G and H be finitely generated groups and suppose that there is a quasi-isometric embedding $G \to H$. Does this imply that there is a quasi-isometric embedding $H \to G$?

b) For which finitely generated groups G are G and $\mathrm{Hom}(G, \mathbb{Z}/2)$ quasi-isometric?

You can hand in your solutions during the exercise classes.