LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Geometric Group Theory

Sheet 7

Exercise 1. Let $c, c' \in \mathbb{R}_{>0}$ and $b, b' \in \mathbb{R}_{>0}$.

a) Let $c' \geq c$ and $b' \geq b$. Is every (c, b)-quasi-isometric embedding then also a (c', b')-quasi-isometric embedding?

b) Let $c' \leq c$ and $b' \leq b$. Is every (c, b)-quasi-isometric embedding then also a (c', b')-quasi-isometric embedding?

Exercise 2.

a) Are the metric spaces $\mathbb{N}$ and $\mathbb{Z}$ (with respect to the standard metric induced from $\mathbb{R}$) quasi-isometric?

b) Are the metric spaces $\mathbb{Z}$ and $\{n^3 | n \in \mathbb{Z}\}$ (with respect to the standard metric induced from $\mathbb{R}$) quasi-isometric?

Exercise 3.

a) Show that every map at finite distance from a quasi-isometric embedding is a quasi-isometric embedding.

b) Show that every map at finite distance from a quasi-isometry is a quasi-isometry.

Exercise 4. Let X, Y, Z be metric spaces and let $f, f' : X \to Y$ be maps that have finite distance from each other.

a) Show that for all maps $g : Z \to X$ the compositions $f \circ g$ and $f' \circ g$ have finite distance from each other.

b) Show that if $g : Y \to Z$ is a quasi-isometric embedding, then $g \circ f$ and $g \circ f'$ also have finite distance from each other.

(please turn)

Conclude the following:

c) Compositions of quasi-isometric [bilipschitz] embeddings are quasi-isometric [bilipschitz] embeddings.

d) Compositions of quasi-isometries [bilipschitz equivalences] are quasi-isometries [bilipschitz equivalences].

You can hand in your solutions during the exercise classes.