LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Summer term 2018

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Geometric Group Theory

Sheet 4

Exercise 1.

a) Is there a free isometric action of $\mathbb{Z}_2$ on $\mathbb{R}$?

b) Is there a free isometric action of $\mathbb{Z}_2$ on $\mathbb{R} \setminus \{0\}$?

c) Is there a free isometric action of $\mathbb{Z}_3$ on $\mathbb{R}$?

d) Is there a free isometric action of $\mathbb{Z}^2$ on $\mathbb{R}$?

Exercise 2. Let $n \in \mathbb{N}$, let $a \in \mathrm{GL}(n, \mathbb{R})$ and let $G := \langle a \rangle_{GL(n,\mathbb{C})}$.

a) Suppose that G acts freely by matrix multiplication on $\mathbb{R}^n \setminus \{0\}$. Show that G then also acts freely by matrix multiplication on $\mathbb{C}^n \setminus \{0\}$.

b) Let $n \geq 2$. Give an example of a non-trivial element $a \in \mathrm{SL}(n, \mathbb{R})$ such that G acts freely on $\mathbb{R}^n \setminus \{0\}$.

Exercise 3. Prove (without using the characterisation of free groups in terms of free actions on trees) that every action of a finite group on a non-empty tree has a global fixed point (i.e., a vertex or an edge on which all group elements act trivially).

Exercise 4.

a) Is every action of a free group on a tree free?

b) Suppose that a free group acts freely on a graph X. Is X then a tree?

You can hand in your solutions during the exercise classes.