LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

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Prof. D. Kotschick
G. Placini

Geometric Group Theory

Sheet 3

Exercise 1. Let $X = (V, E)$ be a finite connected graph with $V \neq \emptyset$. Show that X is a tree if and only if

$$|E| = |V| - 1.$$

Exercise 2. Sketch the Cayley graph of $\mathbb{Z}_2 * \mathbb{Z}_5$ with respect to some finite generating set of your choice.

Exercise 3.

Is there a finitely generated group G with a finite generating set S such that the corresponding Cayley graph $\mathrm{Cay}(G, S)$ is a tree all of whose vertices have degree 3?

Exercise 4.

Let F be a free group of rank 2 and let $S \subset F$ be a free generating set. Show that the Cayley graph $\mathrm{Cay}(F, S)$ admits uncountably many graph automorphisms.

You can hand in your solutions during the exercise classes.