

LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT



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## Geometric Group Theory

Sheet 1

**Exercise 1.** Show that the quotient group A = G/[G,G] of a group G by its commutator subgroup is abelian.

**Exercise 2.** The index [G:H] of a subgroup  $H \subset G$  is the cardinality of the set of cosets G/H.

- a) Let  $H, K \subset G$  be subgroups of finite index. Show that  $H \cap K$  also has finite index in G.
- b) Let  $H \subset G$  be a subgroup and  $S \subset G$  be a set of representatives of the cosets  $\{g \cdot H | g \in G\}$ . Show that

$$\bigcap_{g \in G} g \cdot H \cdot g^{-1} = \bigcap_{g \in S} g \cdot H \cdot g^{-1}.$$

c) Let  $H \subset G$  be a subgroup of finite index. Show that there exists a normal subgroup  $N \subset G$  of finite index with  $N \subset H$ .

**Exercise 3.** Determine the groups  $Out(\mathbb{Z})$  and  $Out(\mathbb{Z}/p\mathbb{Z})$  for a prime  $p \in \mathbb{N}$ .

**Exercise 4.** Let  $3 \le n \in \mathbb{N}$  and let  $X_n \subset \mathbb{R}^2$  be a regular *n*-gon. Describe the group  $D_n$  of symmetries of  $X_n$ . How many elements does  $D_n$  contain?