Summer term 2017

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Topology II

Sheet 9

Exercise 1. Show that if M is a compact contractible n-manifold then its boundary ∂M is a homology $(n-1)$-sphere, that is, $H_i(\partial M;\mathbb{Z}) \simeq H_i(S^{n-1};\mathbb{Z})$ for all i.

Exercise 2. Prove that $\sigma(M\sharp N) = \sigma(M) + \sigma(N)$ for any compact, connected oriented manifolds M and N.

Exercise 3.

(a) Show that there exist a continuous surjective map $\pi : \mathbb{C}\mathrm{P}^{2m+1} \to \mathbb{H}\mathrm{P}^m$ such that $\pi^{-1}(x)$ is homeomorphic to $\mathbb{C}\mathrm{P}^1$ for all $x \in \mathbb{H}\mathrm{P}^m$.

(b) Use point (a) to conclude that the odd dimensional complex projective space $\mathbb{C}\mathrm{P}^{2m+1}$ is bordant to the empty manifold.

Exercise 4. For any embedding $i : S^k \to S^n$ compute the reduced homology groups $\tilde{H}_i(S^n \backslash i(S^k);\mathbb{Z})$ for all i.

Hand in: Tuesday, July 11th, during the exercise class.