Summer term 2017

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Topology II

Sheet 8

Exercise 1. Show that $H_c^n(X \times \mathbb{R}; G) \simeq H_c^{n-1}(X; G)$ for all n.

Exercise 2. Show that the Betti number $b_{2n+1}(M)$ is even for a compact connected orientable manifold M of dimension $4n+2$.

Exercise 3. Let T_i (resp. T^i) be the torsion subgroup of the integral homology group $H_i(M; \mathbb{Z})$ (resp. cohomology group $H^i(M; \mathbb{Z})$) where M is a compact connected orientable manifold.

- Prove that if $\dim(M) = 3$ then $T_i = 0$ for $i \neq 1$, $T^i = 0$ for $i \neq 2$ and $T_1 \simeq T^2$.
- What is the analogous statement for $\dim(M) = 4$?

Exercise 4. Compute the cup product structure in $H^*((S^2 \times S^8)\sharp(S^4 \times S^6); \mathbb{Z})$, and in particular show that the only nontrivial cup products are those given by the projections on summands, e.g. $\pi_1 : (S^2 \times S^8)\sharp(S^4 \times S^6) \to S^2 \times S^8$.
How does it generalize to a connected sum of $S^i \times S^{n-i}$'s for fixed n and varying i?

Hand in: Tuesday, July 4th, during the exercise class.