Summer term 2017 Prof. D. Kotschick
G. Placini

Topology II

Sheet 6

Exercise 1. Describe $H^*(\mathbb{CP}^\infty/\mathbb{CP}^1;\mathbb{Z})$ as a ring with finitely many multiplicative generators. How does this ring compare with $H^*(S^6\times\mathbb{HP}^\infty;\mathbb{Z})$?
[Hint: You may use Künneth formula to compute $H^*(S^6\times\mathbb{HP}^\infty;\mathbb{Z})$]

Exercise 2. For a fixed coefficient field F, define the **Poincaré series** of a space X to be the formal power series $p(t)=\sum_i a_i t^i$ where a_i is the dimension of $H^i(X;F)$ as a vector space over F, assuming this dimension is finite for all i. Show that $p(X\times Y)=p(X)p(Y)$ and compute the Poincaré series for S^n, $\mathbb{RP}^n$, $\mathbb{RP}^\infty$, $\mathbb{CP}^n$ and $\mathbb{CP}^\infty$.

Exercise 3. Show that the splitting of the short exact sequence in the Universal Coefficient Theorem for homology is not natural. Namely exhibit a map $f: X\to Y$ and a group G such that the induced maps are trivial in homology with integer coefficients but nontrivial in homology with coefficients in G.

Exercise 4. Use the Universal Coefficient Theorem to show that if $H_*(X;\mathbb{Z})$ is finitely generated, so the Euler characteristic $\chi(X)=\sum_n(-1)^n\mathrm{rank}H_n(X;\mathbb{Z})$ is defined, then we have $\chi(X)=\sum_n(-1)^n\mathrm{rank}H_n(X;F)$ for any coefficient field F.

Hand in: Tuesday, June 20th, during the exercise class.