Summer term 2017

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Topology II

Sheet 3

Exercise 1. Let X be a Moore space $M(\mathbb{Z}_m, n)$ obtained by attaching a cell e^{n+1} to S^n by a map of degree m.

1. Show that the quotient map $X \to X/S^n = S^{n+1}$ induces the trivial map on $\tilde{H}_i(-;\mathbb{Z})$ for all i, but not on $H^{n+1}(-;\mathbb{Z})$. Deduce that the splitting in the universal coefficient theorem for cohomology cannot be natural.

2. Show that the inclusion $S^n \hookrightarrow X$ induces the trivial map on $\tilde{H}^i(-;\mathbb{Z})$ for all i, but not on $H_n(-;\mathbb{Z})$.

Exercise 2. Compute the cohomology groups of $T^2 = S^1 \times S^1$, $\mathbb{R}\mathrm{P}^2$ and the Klein bottle with coefficients in $\mathbb{Q}$ using simplicial cohomology.

Exercise 3. Determine the cup product $\smile\colon H^1(T^2;\mathbb{Q}) \times H^1(T^2;\mathbb{Q}) \to H^2(T^2;\mathbb{Q})$.

Exercise 4. Let X and Y be topological spaces with $x \in X$ and $y \in Y$ such that (X, x) and (Y, y) are good pairs. Show that

$$\alpha \smile \beta = 0 \in \tilde{H}^{k+l}(X \vee Y; G)$$

for $\alpha \in \tilde{H}^k(X;G)$ and $\beta \in \tilde{H}^l(Y;G)$ where we use the isomorphism $H^i(X \vee Y;G) \simeq H^i(X;G) \oplus H^i(Y;G)$.

Hand in: Tuesday, May 16th, during the exercise class.