LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

Summer term 2017

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Topology II

Sheet 2

Exercise 1. Let X be a topological space and G an Abelian group. Regarding a cochain $\varphi \in C^1(X;G)$ as a function from paths in X to G, show that if φ is a cocycle and $f, g : [0,1] \to X$ are paths, then

1. $\varphi(f \cdot g) = \varphi(f) + \varphi(g)$ where $f(1) = g(0)$ and $f \cdot g$ is the concatenation of f and g,
2. φ takes the value 0 on constant paths,
3. $\varphi(f) = \varphi(g)$ if $f \simeq g$,
4. φ is a coboundary if and only if $\varphi(f)$ depends only on the endpoints of f, for all f.

Exercise 2.

1. Compute the cohomology groups $H^i(S^n;G)$ for $i, n \geq 0$ and G an arbitrary Abelian group.
2. Show that if $f : S^n \to S^n$ has degree d then $f^* : H^n(S^n;G) \to H^n(S^n;G)$ is multiplication by d.

Exercise 3. Given two topological spaces X, Y and an Abelian group G, compute the cohomology groups $H^n(X \vee Y;G)$ of the wedge sum $X \vee Y$.

Exercise 4. Compute the cohomology groups $H^i(\mathbb{R}\mathrm{P}^n;G)$ for all i and n with arbitrary coefficients. [You can assume the homology of $\mathbb{R}\mathrm{P}^n$]

Hand in: Tuesday, May 9th, during the exercise class.