LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
MATHEMATISCHES INSTITUT

Dr. Iosif Petrakis

Winter term 19/20
09.12.2019

Mathematics for Natural Scientists I

Sheet 9

Exercise 1. **(i)** Let the function $f : \mathbb{R} \to \mathbb{R}$, defined by

$$f(x) = \begin{cases} -1 & , x \leq \sqrt{2} \\ 1 & , x > \sqrt{2}. \end{cases}$$

Draw the graph of f and show that f is not continuous at $x_0 = \sqrt{2}$.
[2 points]
(ii) Let the function $g : \mathbb{Q} \to \mathbb{R}$, defined by

$$g(x) = \begin{cases} -1 & , x < \sqrt{2} \\ 1 & , x > \sqrt{2}. \end{cases}$$

Show that g is continuous on $\mathbb{Q}$.
[2 points]

Exercise 2. Let $D \subseteq \mathbb{R}$ and let $f : D \to \mathbb{R}$ be continuous on D. Show the following:
(i) The function $f_1 : D \to \mathbb{R}$, defined by

$$f_1(x) = |f(x)|,$$

for every $x \in \mathbb{R}$, is continuous on D.
[1 point]
(ii) The function $f_2 : D \to \mathbb{R}$, defined by

$$f_2(x) = -f(x)^2,$$

for every $x \in \mathbb{R}$, is continuous on D.
[1 point]
(iii) The function $f_3 : D \to \mathbb{R}$, defined by

$$f_3(x) = \sqrt{2019 f(x)^4 + 2020 f(x)^2},$$

for every $x \in \mathbb{R}$, is continuous on D.
[2 points]

Exercise 3. For every $n \in \mathbb{N}$ let the function $g_n : \mathbb{R} \to \mathbb{R}$, defined by

$$g_n(x) = \frac{nx}{1 + |nx|},$$

for every $x \in \mathbb{R}$.

(i) Draw the graph of the function g_0.

[1 point]

(ii) Determine the values $g_1(-2019)$ and $g_2(2019)$.

[1 point]

(iii) Let the function g, defined by the rule

$$g(x) = \lim_{n \longrightarrow \infty} g_n(x).$$

(a) For which $x \in \mathbb{R}$ is the function g defined?

[1 point]

(ii) Determine the set

$$C(g) = \{x \in \mathbb{R} \mid g \text{ is continuous at } x\}.$$

[1 point]

Exercise 4. Let $a, b \in \mathbb{R}$ such that $a < b$, and let $f : [a, b] \to [a, b]$ be continuous on $[a, b]$. Show that there exists $x_0 \in [a, b]$ such that $f(x_0) = x_0$.

[4 points]

Submission. Wednesday 18. December 2019, in the Exercise-session.

Discussion. Wednesday 18. December 2019, in the Exercise-session.