Dr. Iosif Petrakis

Winter term 19/20
02.12.2019

Mathematics for Natural Scientists I

Sheet 8

Exercise 1. **(i)** Let the function $h_1 : \mathbb{R} \to \mathbb{R}$, defined by

$$h_1(x) = x^2 + 1,$$

for every $x \in \mathbb{R}$. Draw the graph of h_1.
[1 point]
(ii) Let the function $h_2 : \mathbb{R} \to \mathbb{R}$, defined by

$$h_2(x) = -x^2,$$

for every $x \in \mathbb{R}$. Draw the graph of h_2.
[1 point]
(iii) Let the function $h_3 : \mathbb{R} \to \mathbb{R}$, defined by

$$h_3(x) = (x - 1)^2,$$

for every $x \in \mathbb{R}$. Draw the graph of h_3.
[2 points]

Exercise 2. Let the polynomials $p, q : \mathbb{R} \to \mathbb{R}$, defined by

$$p(x) = 2 + 3x + 4x^2,$$

$$q(x) = 3 + x,$$

for every $x \in \mathbb{R}$, respectively.
(i) Determine the composition function $p \circ q$.
[1 point]
(ii) Determine the composition function $q \circ p$.
[1 point]
(iii) Determine the domain of the rational function $R_{pq}(x) = \frac{p(x)}{q(x)}$, and find the value $R_{pq}(-1)$.
[2 points]

Exercise 3. The *ceiling function* $\lceil . \rceil : \mathbb{R} \to \mathbb{R}$ is defined by $x \mapsto \lceil x \rceil$, for every $x \in \mathbb{R}$, where $\lceil x \rceil$ is the unique integer such that

$$\lceil x \rceil - 1 < x \leq \lceil x \rceil.$$

(i) Draw the graph of the ceiling function.

[2 points]

(ii) Find the limits

$$\lim_{x \longrightarrow 0^+} \lceil x \rceil \quad \& \quad \lim_{x \longrightarrow 0^-} \lceil x \rceil.$$

[2 points]

Exercise 4. Let the Dirichlet function $\texttt{Dir} : \mathbb{R} \to \mathbb{R}$, defined by

$$\texttt{Dir}(x) := \begin{cases} 1 & , x \in \mathbb{Q} \\ 0 & , x \in \mathbb{I}, \end{cases}$$

(i) Find a sequence $(\alpha_n)_{n \in \mathbb{N}}$ of reals such that the following conditions are satisfied:
(a) $\alpha_n > 0$, for every $n \in \mathbb{N}$, and
(b) $\alpha_n \in \mathbb{Q}$, for every $n \in \mathbb{N}$, and
(c) $\lim_{n \longrightarrow \infty} \alpha_n = 0$.

[1 point]

(ii) Find a sequence $(\beta_n)_{n \in \mathbb{N}}$ of reals such that the following conditions are satisfied:
(c) $\beta_n > 0$, for every $n \in \mathbb{N}$, and
(d) $\beta_n \in \mathbb{I}$, for every $n \in \mathbb{N}$, and
(e) $\lim_{n \longrightarrow \infty} \beta_n = 0$.

[1 point]

(iii) Determine the following limits

$$\lim_{n \longrightarrow \infty} \texttt{Dir}(\alpha_n) \quad \& \quad \lim_{n \longrightarrow \infty} \texttt{Dir}(\beta_n).$$

[1 point]

(iv) Does the limit

$$\lim_{x \longrightarrow 0^+} \texttt{Dir}(x)$$

exist?

[1 point]

Submission. Wednesday 11. December 2019, in the Exercise-session.

Discussion. Wednesday 11. December 2019, in the Exercise-session.