LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

MATHEMATISCHES INSTITUT

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Winter term 19/20
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Mathematics for Natural Scientists I

Sheet 7

Exercise 1. **(i)** Find the limit

$$\sum_{n=2}^{\infty} \frac{1}{n(n-1)}.$$

[1 point]

(ii) If $x \in \mathbb{R}$ and $n \in \mathbb{N}$, show that

$$(1-x)\left(\sum_{k=0}^{n} x^k\right) = 1 - x^{n+1}.$$

[1,5 points]

(iii) Find $x \in \mathbb{R}$ such that

$$\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^n = x.$$

[1,5 points]

Exercise 2. If $(\alpha_n)_{n \in \mathbb{N}}$ is a sequence of real numbers, show the following implication

$$\sum_{n=0}^{\infty} |\alpha_n| \in \mathbb{R} \;\Rightarrow\; \sum_{n=0}^{\infty} \alpha_n \in \mathbb{R}.$$

[2 points]

(ii)(a) If $k \geq 2$, show that for every $n \geq 1$ it holds

$$\frac{1}{n^k} \leq \frac{1}{n^2} \leq \frac{2}{n(n+1)}.$$

[1 point]

(b) With the use of (a) show that

$$\sum_{n=1}^{\infty} \frac{1}{n^k} \in \mathbb{R}, \qquad k \geq 2.$$

[1 point]

Exercise 3. Find the limit

$$\sum_{n=1}^{\infty} \frac{1}{4n^2-1}.$$

[4 points]
[**Hint**: Use the equality

$$\frac{1}{2n-1} - \frac{1}{2n+1} = \frac{2}{(2n-1)(2n+1)}.]$$

Exercise 4. Let $x, y \in \mathbb{R}$, and let $(\alpha_n)_{n\in\mathbb{N}}$ be a sequence of real numbers, defined by

$$\alpha_n := \begin{cases} x & , n = 0 \\ y & , n = 1 \\ \frac{\alpha_{n-1}+\alpha_{n-2}}{2} & , n \geq 2, \end{cases}$$

Show the following:
(i) If $k \geq 1$, then

$$\alpha_{k+1} - \alpha_k = \Big(-\frac{1}{2}\Big)(\alpha_k - \alpha_{k-1}).$$

[1 point]
(ii) For every $k \in \mathbb{N}$

$$\alpha_{k+1} - \alpha_k = \Big(-\frac{1}{2}\Big)^k (y - x).$$

[1 point]
(iii) If $n \geq 1$, then

$$\alpha_n = x + (y - x) \sum_{k=0}^{n-1} \Big(-\frac{1}{2}\Big)^k.$$

[1 point]
(iv) $\lim_{n\longrightarrow\infty} \alpha_n = \frac{1}{3}(x + 2y)$.
[1 point]

Submission. Wednesday 04. December 2019, in the Exercise-session.

Discussion. Wednesday 04. December 2019, in the Exercise-session.