LUDWIG-
MAXIMILIANS-
UNIVERSITÄT
MÜNCHEN

MATHEMATISCHES INSTITUT

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Mathematics for Natural Scientists I

Sheet 6

Exercise 1. Let $(\texttt{Fib}_n)_{n\in\mathbb{N}}$ be the sequence of Fibonacci numbers.
(i) Show that for every $n \geq 1$

$$\texttt{Fib}_{n+1} \cdot \texttt{Fib}_{n-1} - \texttt{Fib}_n^2 = (-1)^n.$$

[2 points]
(ii) Show that

$$\lim_{n\longrightarrow\infty} \frac{\texttt{Fib}_{n+1} \cdot \texttt{Fib}_{n-1}}{\texttt{Fib}_n^2} = 1.$$

[2 points]

Exercise 2. Let $a, b \in \mathbb{R}$ such that $a > 0$ and $b > 0$. Let the sequence $(\alpha_n)_{n\in\mathbb{N}}$ be defined by

$$\alpha_0 = b,$$

$$\alpha_{n+1} = \frac{1}{2}\left(\alpha_n + \frac{a}{\alpha_n}\right).$$

Show the following:
(i) $\alpha_n > 0$, for all $n \in \mathbb{N}$.
[1 point]
(ii) $\alpha_n^2 \geq a$, for all $n \geq 1$.
[1,5 points]
(iii) $\alpha_{n+1} \leq \alpha_n$, for all $n \geq 1$.
[1,5 points]

Exercise 3. Prove or disprove the following:
(i) If $q \in \mathbb{Q}$, then $\sqrt{2} + q \in \mathbb{Q}$.
[0,5 point]
(ii) If $q \in \mathbb{Q}$, then $q \cdot \sqrt{2} \in \mathbb{Q}$.
[0,5 point]
(iii) $\sqrt{2} \cdot \sqrt{4} \in \mathbb{Q}$.
[1 point]
(iv) For every $a, b \in \mathbb{I}$ it holds that $a + b \in \mathbb{I}$.
[1 point]
(v) For every $a, b \in \mathbb{I}$ it holds that $a \cdot b \in \mathbb{I}$.
[1 point]

Exercise 4. Show that the set $\mathbb{Q}$ of rational numbers does not satisfy the Completeness Axiom.
[**Hint**: Use the sequence

$$\alpha_0 = 1,$$
$$\alpha_{n+1} = \frac{1}{2}\left(\alpha_n + \frac{2}{\alpha_n}\right),$$

in order to prove this.]
[4 points]

Submission. Wednesday 27. November 2019, in the Exercise-session.

Discussion. Wednesday 27. November 2019, in the Exercise-session.